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13. ABSTRACT *(Maximum 200 words)*

The main thrust of research is on defense oriented projects. Significant contributions are made in the area of signal estimation and multitarget tracking. The main problems addressed are design of observations, efficient estimation of target positions almost continuously in time and establishing correct association between the estimates of target positions made at different points of time. A complete package has been developed for angles only tracking using sensor array data, which can be implemented in real time without unduly heavy computations. The method developed is quite general in nature and can be extended to tracking targets in terms of positional coordinates, and for a variety of models for the observations.

On related problems: A unified theory of robust estimation with minimal assumptions is developed. Some problems in probability theory, non-linear time series, bootstrap methodology and shape analysis useful in signal estimation and imaging are investigated. A new line of research is initiated in the use of differential geometric methods in statistical inference.

14. SUBJECT TERMS

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FINAL TECHNICAL REPORT

(Under Contract No. DAAL03-89-K-0139)

August 1990 - August 1992

Description of Research Work

The research work done during the period August 1990 - August 1992, under support from the Army Research Office, contract no. DAAL03-89-K-0139, is reported in technical reports issued by the Center for Multivariate Analysis (CMA). A list of these reports with the abstracts is given in the Appendix. The contents of the present report are as follows:

1. Broad areas of research under report
2. Brief outline of research work
3. Academic activities of the CMA (August 1990 - August 1992)
   - 3.1. Honors received by C. R. Rao, the principal investigator
   - 3.2. Research participants
   - 3.3. Ph.D. degrees awarded to research students
   - 3.4. Participation in conferences in 1992.

The research work was undertaken by a team of senior scientists, post-doctoral research associates and graduate students who were all supported by the grant from the ARO during the period under review. The work was done under the direction of C. R. Rao, the Director of the CMA and the principal investigator.

1. Broad Areas of Research Under Report

The main thrust of research is on signal estimation and multitarget tracking. We have been able to develop a complete package for angles only tracking, which can be implemented in real time without unduly heavy computations. The methods developed are of a general nature and can be extended to tracking targets in terms of positional coordinates, and for a variety of models for the observations.

Research is also undertaken on statistical problems arising in defense oriented projects. A new line of research is initiated on the size and shape analysis of landmark data, which has applications to problems like infrared imaging. An extensive study is made of robust estimation of unknown parameters and testing of hypotheses. Advances have been made in investigating the convergence rates of weighted sums of random variables, which are useful in studying properties of estimates. Analysis of non-linear time series is developed through the estimation of bispectrum. This is useful in estimating the parameters of signals in the presence of noise and for discriminating between deterministic non-linear models and chaotic models.

The use of differential geometric methods in statistical inference is a topic of active research in recent times. A new approach is suggested by introducing a differential geometric structure for a statistical model which is different from the one considered in current research work. This new structure has the potential to solve a wide variety of problems in statistical inference.

2. Brief Outline of Research Work

2.1 Signal Detection, Estimation and Multitarget Tracking

We consider two models for the directions of arrival (DOA) of signals. One is the multiple sinusoidal model

$$y(r) = \sum_{k=1}^{m} \alpha_k e^{jw_k r} + \varepsilon(r), \quad r = 1,...,N. \tag{1.1}$$

Another is the p-array sensor model

$$\boldsymbol{y}(t) = A\boldsymbol{s}(t) + \boldsymbol{\varepsilon}(t), \quad t = 1,...,N$$

where $\boldsymbol{y}(t)$ is a p-vector of observations, $\boldsymbol{s}(t)$ is an unknown m×1 random vector, $\boldsymbol{\varepsilon}(t)$ is a p-vector of white noise and A is a p×m matrix with the r-th row as

$$(e^{-jw_1(r-1)},...,e^{-jw_m(r-1)}), \quad r = 1,...,p. \tag{1.2}$$

The above models refer to signals from m targets measured at close intervals of time during which the targets can be considered to be stationary. The statistical problem in such a case is the estimation of the parameters $w_1,...,w_m$ which provide the directions of arrival (DOA) of signals from the m stationary targets.

If the targets are moving, we can take observations in successive periods of short intervals of time and obtain estimates of the DOA at different time periods. For purposes of tracking the targets, i.e., tracing the trajectories of the different

targets, it is necessary to establish the correct association of the estimated DOA's at different periods. In addition, there are other complications such as missing observations and spurious observations (from clutter).

In earlier work conducted at the CMA, the problems of detection of signals and estimation of the number of signals using information theoretic criteria were investigated. [See Chapter 9 in Advances in Spectrum Analysis and Array Processing, Vol. 11, Edited by S. Haykin.] The information theoretic criteria are valid in large samples. In [91-33], the small sample properties are investigated through extensive simulation studies. During the last three years, research was concentrated on the estimation of signals and multitarget tracking.

In [90-48], the asymptotic properties of the m.l. (maximum likelihood) estimates of the w-parameters under the model (1.1) are studied. It is shown that the m.l. estimates attain the Cramer-Rao lower bound for the asymptotic variance-covariance matrix. A numerical algorithm is developed in [91-34] to obtain the m.l. estimates under the model (1.1).

Closed form solutions for the estimation of the w-parameters in models (1.1) and (1.2) are obtained by computing the roots of polynomial equations with coefficients estimated from the observations. They have a certain advantage over other algorithms like MUSIC which are basically search methods. Details are given in Chapter 9 of Advances in Spectrum Analysis and Array Processing, Vol. 11, edited by S. Haykin, in [91-17] and in the Thesis: Estimation of Directions of Arrival in Signal Processing Models by the graduate student Nandini Kannan.

The main thrust in the research work done during the last twelve months is on multitarget tracking. The statistical problems in multitarget tracking are

formulated and discussed in [92-19]. The basic algorithms in the case of angles only tracking are developed in [91-14, 91-27, 91-29, 92-04, 92-18].

The success of our approach is due to several improvements made over the previous results. First, we use the m.l. method to estimate the DOA's and thereby obtain estimates with maximum possible accuracy, nearly achieving Cramer-Rao lower bound. Second, inaccuracies in data association are minimized by estimating the DOA's in such a way that the association between the estimates made at different time points is maintained. Third, appropriate changes are made in estimation when the targets cross or come close together to increase the efficiency of estimates and enable discrimination between the DOA's which differ by small amounts. Extensive simulation studies indicate that our methods perform well for any number of targets and even under low signal to noise ratios.

The methods developed by us are quite general and can be extended to tracking targets in terms of position coordinates, and for a variety of models for the observations.

2.2 Robust Estimation

A unified theory is developed for robust or M-estimation of parameters and testing of hypotheses in a linear model

$$y_i = x_i'\beta + \varepsilon_i, \quad i = 1,...,n$$

in a series of papers. In M-estimation, one chooses a discrepancy function ρ and obtains estimates of parameters by minimizing the function

$$\sum_{i=1}^{n} \rho(y_i - x'_i\beta).$$

There are numerous papers on the subject based on different choices of the ρ function. In each case, the results are established under heavy assumptions. In our work, first a general theory is developed taking ρ to be a convex function. In such a case, results can be proved using a minimal set of assumptions. This covers a wide variety of situations including the LAD (least absolute deviation) method. The results are given in [90-35, 90-38, 90-42, 91-02, 91-03, 91-07, 91-15, 91-37, 92-23]. Multivariate extensions are reported in [90-39, 91-04, 91-22].

The theory is generalized in [92-14] to cover a wide variety of situations by considering ρ as the difference of two convex functions. Most of the ρ functions considered by the previous authors have approximately the same form as the difference of two convex functions. The advantage in choosing such a ρ function is that the results can be proved under much milder conditions than those used by the previous authors. A further generalization to cover every possible case is considered in [92-17] where the ρ function is taken to be non-increasing in $(-\infty,0]$ and non-decreasing in $[0,\infty)$.

2.3 Probability Theory

Some results are obtained on strong limit theorems in general probability spaces. A moderate deviation result is derived for stochastic processes with stationary and independent increments with an explicit computation of the rate of convergence. The results are reported in [91-10, 91-11].

2.4 Bootstrap

Bootstrap methodology was explored in finding the sampling distributions of Student's t using the mean absolute deviation instead of the standard deviation in the denominator and of the statistic based on Rao's quadratic entropy for testing differences between populations in one way classification. These are non-standard situations where bootstrap has to be applied on modified statistics. The results are reported in [91-06]. Applications of bootstrap inference are reviewed in a survey paper [92-16].

2.5 Shape Theory and Image Analysis

Study of objects by their shape and construction of shape from a digital image have important applications in biology, biomedical work and signal processing. Much of the work previously done in this area was empirical in nature. A systematic attempt is being made at the Center for Multivariate Analysis to develop a suitable theory for size and shape distributions and to provide numerical algorithms for applications in practical research work.

There are fundamental differences between classical multivariate analysis and shape theory. Shape theory requires new results in multivariate distribution theory. A complete set of results in size-and-shape densities and shape densities in any number of dimensions is derived. They include the results for planar figures worked out by various authors. The scope of multivariate analysis has been considerably extended by the use of these new models for shape distributions. Some application of these results to problems in biology and reconstruction of shapes from digital images have been considered.

The results are reported in [91-13, 91-19, 91-20 and 91-28].

2.6 Differential Geometric Methods in Statistical Inference

Differential geometric methods in statistical inference were first introduced by C. R. Rao in 1945. It is only recently that interest in this area has been revived and important contributions have been made by a number of authors. References to recent literature are given in [92-24].

The general approach in all these papers is based on the original formulation given by C. R. Rao, by considering a space of parameters of a family of distribution functions and imposing on it a Riemannian structure with the Fisher information matrix providing the matrix for the quadratic differential metric.

In [92-24], a new differential geometric structure for a statistical model is proposed. If $\theta = (\theta_1,...,\theta_p)$ represents a p-dimensional parameter and $f(x,\theta)$, the probability density for given θ, then one may consider the manifold

$$\{(\theta_1,...,\theta_p,\theta_{p+1}):\ \theta_{p+1} = \log f(x,\theta)\}$$

which is a subset of R^{p+1}. Based on this setup, the Riemannian differential metric can be computed as

$$\sum\sum (b_{rs} + i_{rs}) d\theta_r d\theta_s$$

where $I = (i_{rs})$ is Fisher information matrix and $B = (b_{rs})$ is any positive definite matrix. In the earlier work, the metric is defined using only the matrix I. The introduction of the matrix B considerably enlarges the scope of differential geometric

methods in statistical inference. Details such as the definition of affine connections and applications to some well-known distributions are given in [92-24].

The concept of SOE (second order efficiency or information loss in large samples) in statistical estimation, by which several competing estimates can be compared for higher order efficiency, was introduced by C. R. Rao in 1961. The discussion by C. R. Rao and later writers on the subject was confined to the estimation of one unknown parameter. The concept of SOE (or information loss) is extended to the multiparametric case in [92-27].

In [92-25], the concept of Gauss curvature of a statistical model is developed and a natural connection between Gauss curvature and Fisher information matrix is established. The concept of mean curvature is introduced and its use in statistical theory is discussed. In particular, the concept of mean curvature is used in studying the asymptotic properties of the generalized Bayes estimators in the location parameter family of distributions. Details are given in [92-26].

Differential geometric methods in statistical inference is an important topic of current research in statistics. The work reported above breaks a new ground and solves some outstanding problems.

3. Academic Activities of the CMA (August 1990 - August 1992)

3.1 Honors received by C. R. Rao, the principal investigator

1990:	Honorary D.Sc.	Colorado State University Fort Collins, U.S.A.
1991:	Honorary Doctoris	University of Poznan Poznan, Poland

1991	Honorary D.Sc.	University of Hyderabad Hyderabad, India

3.2 Research Participants

The following senior scientists and post-doctoral research associates were supported by the grant by ARO under contract no. DAAL03-89-K-0139:

Senior Scientists:	Z. D. Bai, L. C. Zhao, T. Subba Rao and K. V. Mardia
Post-doctoral Research Associates:	D. Kundu, N. Kannan, Min Deng, Bin Zhou and C. R. Sastry.

3.3 Ph.D. Degrees Awarded to Research Students

The following graduate students working under the supervision of C. R. Rao received Ph.D. degrees during the period under review.

1990	Min Deng	Differential Geometry in Statistical Inference
	Zhijun Liu	Some Contributions to Nonparametric nd Robust Estimation
1992	Nandini Kannan	Estimation of Directions of Arrival in Signal Processing Models

3.4 Participation in Conferences in 1992

The principal investigator, C. R. Rao participated in the following conferences to present papers on signal detection and multitarget tracking.

1. Seventh International Conference on Multivariate Analysis, 5-9 May 1992, The Pennsylvania State University, University Park, PA.
   Invited Talk: Some recent contributions to multitarget tracking.

2. Tenth Army Conference on Applied Mathematics and Computing, 16-19 June 1992.
   Invited Paper: Current trends of research in statistics: Small sample asymptotics, resampling techniques and robustness.

3. Multi-Sensor Tracking/Survivable Communications and Mathematical Methods and Algorithms: SDIO/IST Program Review, 13-14 July 1992.
   Invited Paper: Statistical problems in identification, location estimation and tracking of multiple targets using multisensor data.

4. Fifth Purdue University Symposium on Statistical Decision Theory and Related Topics, 14-19 June 1992
   Invited Paper: Some statistical problems in multitarget tracking.

APPENDIX

Titles with abstracts of technical reports
on research supported by
grant DAAL03-89-K-0139

(January 1990 - December 1990)

90-11 Debasis Kundu. Asymptotic Properties of the Complex Valued Non-Linear Regression Model, April 1990 (Army).

The non-linear regression model, when the parameters are complex valued is considered here. Jenrich (1969) considered the non-linear regression model when the parameters are real valued. He first rigorously proved the existence of the least square estimator and showed its consistency and asymptotic normality. In this paper we generalize the idea for the complex parameters case. Large sample properties of the proposed estimator has been studied.

90-12 Debasis Kundu. Estimating the Parameters of Exponential Signals, April 1990 (Army).

A new method is proposed for estimating the parameters of exponential model when the data are complex valued. Osborne (1975) has proposed a method to estimate the parameters of exponential model when the data are real valued. The same idea has been generalized to the complex parameters case.

90-14 Gutti Jogesh Babu and Z. D. Bai. Edgeworth Expansions for EIV, April 1990 (Army).

Edgeworth expansions for sums of independent but not identically distributed multivariate random vectors are established. The results are applied to get valid Edgeworth expansions for estimates of regression parameters in linear errors-in-variable models. The expansions for studentized versions are also developed. Further, Edgeworth expansions for the corresponding bootstrapped statistics are obtained. Using these expansions, the bootstrap distribution is shown to approximate the sampling distribution of the studentized estimators, better than the classical normal approximation.

90-17 Gutti Jogesh Babu and Z. D. Bai. Edgeworth Expansions of a Function of a Sample Means Under Minimal Moment Conditions and Partial Cramér's Condition, July 1990 (Army).

A wide class of statistics can be expressed as smooth functions of sample means of random vectors. Edgeworth expansions of such statistics are generally obtained under Cramér's condition. In many

practical situations, like in the case of ratio statistics, only one of the components of the random vector satisfies the Cramér's condition, while the rest do not. Edgeworth expansions are established under partial Cramér's condition. Further the conditions on the moments are relaxed to the minimum needed to define the expansions.

90-22 György Terdik and Laurie Meaux. The Exact Bispectra for Bilinear Realizable processes with Hermite Degree 2, July 1990 (Army).

In this paper the Hermite degree 2 stationary bilinear model is considered which is built up by the first and second order Hermite polynomial of a Gaussian white noise process. The exact spectrum and bispectrum is constructed in terms of the transfer functions of the model. Some examples are added to highlight the hugh variety of possible spectra.

90-23 Kwok-Wai Tam and Yuehua Wu. On Rates of Convergence of Efficient Detection Criteria in Signal Processing when the Noise Covariance Matrix is Arbitrary, July 1990 (Army).

Zhao, Krishnaiah, and Bai proposed certain information theoretic criteria for detection of the number of signals under an additive model in presence of noise with arbitrary covariance matrix. It was shown that these criteria are strongly consistent even when the underlying distribution is not necessarily Gaussian. Upper bounds on the probabilities of error detection are obtained in this paper.

90-24 Z.D. Bai, C. Radhakrishna Rao and Lu Zhang. Tracking the Directions of Arrivals of Signals from Moving Targets PART 1: A Review and some Improved Algorithms, July 1990 (Army)

In a recent paper, Sastry, Kamen and Simaan (1990) suggested a new approach to tracking angles-of-arrival of moving targets. The targets are modeled as signal sources that continuously emit narrow band signals which impinge on an array of sensors. In this paper several improvements are made in the algorithm of Sastry *et al* to provide more efficient and stable estimates of the trajectories of the moving objects.

90-25 C. Radhakrishna Rao and Nandini Kannan. On Modified EVLP Method for Estimating Superimposed Exponential Signals, July 1990 (Army).

We consider the multiple sinusoidal model for signals and provide a modified EVLP (equivariant linear prediction) method when the damping factor is not present. The solution in the general case depends on the roots of a polynomial equation whose coefficients are estimated from observed data. When the damping factor is absent, the

coefficients exhibit a certain symmetry. A satisfactory algorithm is found for the estimation of the coefficients under the symmetry conditions.

90-35 C. Radhakrishna Rao and L. C. Zhao. Linear Representation of M-estimates in Linear Models, September 1990 (Air Force and Army).

Consider the linear regression model, $y_i = x_i'\beta+e_i$, $i = 1,\ldots,n$, and an M-estimate $\hat{\beta}$ of β obtained by minimizing $\Sigma\rho(y_i-x_i'\beta)$ where ρ is a convex function. Let $S_n = \Sigma x_i x_i'$ and $r_n = S_n^{1/2}\hat{\beta} - S_n^{-1/2}\Sigma x_i h(e_i)$ where, with a suitable choice of $h(\cdot)$, the expression $\Sigma x_i h(e_i)$ provides a linear representation of $\hat{\beta}$. Bahadur (1966) obtained the order of r_n as $n \to \infty$ when β is a one dimensional location parameter representing the median, and Babu (1989) proved a similar result for the general regression parameter estimated by the LAD (least absolute divisions) method. We obtain the stochastic order of r_n as $n \to \infty$ for a general M-estimate as defined above, which agrees with the results of Bahadur and Babu in the special cases considered by them.

90-38 Z. J. Liu and Y. Q. Yin. A Note on Bahadur's Representation for the Two Sample Case, October 1990 (Air Force and Army).

In this paper the asymptotic distribution of the two sample problem on Bahadur's representation is investigated. The problem concerns the structure and the relationship of two individual sample medians and their global sample median. It is shown that the global sample median can be represented as linear transform of the two individual sample medians plus a remainder of order $O_p(n^{-3/4}(\log\log n)^{3/4})$. Under certain standardization, the asymptotic normality of the remainder and the asymptotic independence between the remainder and the two individual sample medians are also obtained.

90-39 Z. D. Bai, C. Radhakrishna Rao and L. C. Zhao. MANOVA Type Tests under M-Theory for the Standard Multivariate Linear Model, October 1990 (Air Force and Army).

We provide the M-theory for the standard multivariate linear model $Y = XB + E$, where Y is $n\times p$ matrix of observations, X is $n\times m$ design matrix, B is $m\times p$ matrix of unknown parameters and E is $n\times p$ matrix of errors with the row vectors independently distributed. Two test criteria based on the roots of determinantal equations are proposed for testing linear hypotheses of the form $P'B = C_0$, where P is a matrix

of rank q. The tests are similar to those considered in MANOVA using least squares techniques. One of them is the Wald type statistic and another is the Rao's score type statistic. The asymptotic distributions of these test statistics are derived. Consistent estimates of nuisance parameters are obtained for use in computing the test statistics.

The M-method of estimation considered is the minimization of $\Sigma\rho(e_i)$, where ρ is a convex function and e_i is the i-th row vector in (Y-XB). All results are derived under a minimal set of conditions.

90-42 L. C. Zhao, C. Radhakrishna Rao and X. R. Chen. A Note on the Consistency of M-estimates in Linear Models, December 1990 (Air Force and Army).

Weak consistency of the M-estimate of the regression parameters in a general linear model is established under the condition $(X_n'X_n)^{-1} \to 0$ as $n \to \infty$, where X_n is the design matrix for the first n observations. The M-estimate is obtained by minimizing the sum of $\rho(\epsilon_i)$, $i = 1,\ldots,n$, where ρ is a convex function satisfying some minimal regularity conditions, and ϵ_i is the i-th residual.

90-43 Deli Li, M. Bhaskara Rao and Xiangchen Wang. On the Strong Law of Large Numbers and the Law of the Logarithm for Weighted Sums of Independent Random Variables with Multidimensional Indices, December 1990 (Air Force and Army)

Let $\{X, X_{\bar{n}};\ \bar{n} \in \mathbb{N}^d\}$ be a field of independen t identically distributed real random variables, $0 < p < 2$ and $\{a_{\bar{n},\bar{k}};\ (\bar{n},\bar{k}) \in \mathbb{N}^d \times \mathbb{N}^d\}$ a double array of real numbers. Under the minimal condition that $\sup_{\bar{n},\bar{k}} |a_{\bar{n},\bar{k}}| < \infty$, we show that $|\bar{n}|^{-1/p}\sum_{\bar{k}\le\bar{n}} a_{\bar{n},\bar{k}} X_{\bar{k}} \to 0$ a.s. as $|\bar{n}| \to \infty$ if and only if $E(|X|^p(L|X|)^{d-1}) < \infty$ provided $d \ge 2$. In the above, if $1 \le p < 2$, the random variables are needed to be centered at the mean. By establishing a certain law of logarithm, we show that the Law of the Interated Logarithm fails for the weighted sums $\sum_{\bar{k}\le\bar{n}} a_{\bar{n},\bar{k}} X_{\bar{k}}$ under the conditions $EX = 0$, $EX^2 < \infty$ and $E(X^2(L|X|)^{d-1}/L_2|X|) < \infty$ for almost all bounded families $\{a_{\bar{n},\bar{k}};\ (\bar{n},\bar{k}) \in \mathbb{N}^d \times \mathbb{N}^d\}$ of numbers.

90-44 Li Deli, M. Bhaskara Rao, Jiang Tiefeng and Wang Xiangchen. Complete Convergence and almost sure Converges of Weighted Sums of Random Variables, December 1990 (Air Force and Army).

For each $n \geq 1$, let $\{X_{nk}; -\infty < k < \infty\}$ be sequence of independent real random variables. We provide some very relaxed conditions which will guarantee complete convergence of the sequence $\{\sum_{k=-\infty}^{\infty} X_{nk}; n \geq 1\}$ to zero, i.e., $\sum_{n\geq 1} P\{|\sum_{k=-\infty}^{\infty} X_{nk}| \geq \epsilon\} < \infty$ for every $\epsilon > 0$. This result is used to establish some results on complete convergence for weighted sums of independent random variables. The main ideas are that we exploit a certain maximal inequality and devise an effective way of splitting the weights into several classes in order to control their magnitudes. Some results in the literature follow as special cases of results here. Particularly, we obtain an improvement of Chow's (1966) result as well as a sharpening of the result for "almost sure convergence" to "complete convergence" in the main result of Thrum (1987) under weaker conditions for weights. The general result on complete convergence is also used to derive a result on almost sure convergence from which a result of Deniel and Derriennic (1988) follows.

90-47 T. Subba Rao. Analysis of Nonlinear Time Series (and Chaos) by Bispectral Methods, December 1990 (Air Force and Army).

Second order spectra have played a very significant role in the analysis of linear (Gaussian) time series. When the series is non Gaussian (and nonlinear) it is important to study higher order spectra. Here we briefly consider the estimation of the bispectrum, and discuss the usefulness of this function in several situations, for example, when estimating the parameters of the signal in the presence of noise, for discriminating "deterministic" nonlinear models and chaotic models etc. We also consider bilinear models which were introduced recently in time series literature, and discuss their estimation and study the forecasts obtained in a specific example. Lastly we point out how this model can be generalised to deal with nonlinear long range dependence.

90-48 C. Radhakrishna Rao and L. C. Zhao. Asymptotic Behavior of Maximum Likelihood Estimates of Superimposed Exponential Signals, December 1990 (Army).

In this paper are derived the asymptotic properties of the maximum likelihood estimates (m.l.e.) of the unknown parameters $(\alpha_1,\ldots,\alpha_p)$, $(\omega_1,\ldots,\omega_p)$ and σ^2 in the superimposed exponential model for signals

$$y_t = \sum_{k=1}^{p} \alpha_p \exp(it\,\omega_k) + \epsilon_t$$

$$t = 0,1,\ldots,n-1$$

where σ^2 is the variance in the complex normal distribution of ϵ_t. It is shown that the m.l.e.'s of the parameters attain the Cramer-Rao lower bound for the asymptotic covariance matrix.

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91-02 Zhijun Liu. NONPARAMETRIC ESTIMATES OF THE NUISANCE PARAMETER IN LAD TESTS, January 1991 (*Air Force and Army*)

The L_1-norm method in linear regression model has attracted considerable attention in recent years. Some L_1-norm based techniques for testing linear hypothesis have been proposed and investigated. Unfortunately, a nuisance parameter involved in those test statistics has not been estimated effectively yet. We propose several nonparametric estimates in this paper and prove the strong consistency. Our methods are based on the density estimation method developed by Rosenblatt (1956), Parzen (1962) and others and the conditional density estimation developed by Zhao and Liu (1985)

91-03 C. Radhakrishna Rao and L. C. Zhao. ON THE CONSISTENCY OF M-ESTIMATE IN A LINEAR MODEL OBTAINED THROUGH AN ESTIMATING EQUATION, January 1991 (*Air Force and Army*)

We consider the linear model $y_i = x_i'\beta + e_i$, $i = 1,\ldots,n$ and an estimating equation of the form $\psi(y_1-x_1'\beta)x_1+\ldots+\psi(y_n-x_n'\beta)x_n = 0$ and prove the consistency of the estimator β under some mild conditions on ψ. The conditions imposed are much weaker than those assumed in earlier work by other authors.

91-04 Z. D. Bai, C. Radhakrishna Rao and L. C. Zhao. WEAK REPRESENTATION OF M-ESTIMATES IN MULTIVARIATE LINEAR MODELS, May 1991 (*Air Force and Army*)

Let $Y_i = X_i'\beta + e_i$, $i = 1,\ldots,n,\ldots$ be a p-variate regression model, where e_i are independent p-vector random variables having a common distribution function F. Consider the problem of estimating β by minimizing $\rho(Y_1-X_1'\beta) + \ldots + \rho(Y_n-X_n'\beta)$ where ρ is a convex function. We obtain a weaker form of Bahadur representation of the estimator $\hat{\beta}$ so obtained under a minimal set of conditions.

91-06 Z. J. Liu and C. Radhakrishna. ASYMPTOTIC DISTRIBUTION OF STATISTICS BASED ON QUADRATIC ENTROPY AND BOOTSTRAPPING, May 1991 (*Air Force and Army*)

We investigate the asymptotic properties of some statistics based on quadratic entropy (QE), a notion introduced by Rao (1982) as a general measure of variability in a population. Sample QE based on iid observations from a distribution function is defined and its asymptotic distribution is derived. The use of QE in the place of variance in analysis of diversity of cross classified data provides a generalization of ANOVA. We call this method ANOQE (Analysis of Quadratic Entropy). The ANOQE statistic for testing equality of populations based on one way classified data is constructed and its asymptotic and bootstrap distribution are derived.

91-07 C. Radhakrishna Rao and L. C. Zhao. APPROXIMATION TO THE DISTRIBUTIONS OF M-ESTIMATES IN LINEAR MODELS BY RANDOMLY WEIGHTED BOOTSTRAP, June 1991 (*Air Force and Army*)

We consider the M-estimation of regression parameters in the linear model by minimizing the sum of convex functions of residuals. In earlier papers (see for instance Bai, Rao and Wu (1991) and Yohai and Maronna (1979)), the asymptotic normality of the M-estimator was established. In this paper we discuss the method of Bayesian bootstrap to derive the approximate distribution of the M-estimator. Bayesian bootstrap or the random weighting method was developed by Rubin (1981), Lo (1987), Weng (1989), Zheng (1987) and Tu and Zheng (1987) with reference some statistics such as the sample mean. We extend these results to the general regression problem.

91-09 Raja Velu and M. Bhaskara Rao. ON THE NON-NULL DISTRIBUTION OF SOME ESTIMATORS OF INTERCLASS CORRELATION, June 1991 (*Air Force* and *Army*)

In the context of familial data consisting of measurements on mothers and their offsprings, various estimators have been proposed for interclass correlation, i.e., correlation between mother and offspring, in the literature. In this paper, we derive the non-null distribution of some of these estimators under equal sib-ship size.

91-10 Xiangchen Wang, M. Bhaskara Rao and Deli Li. SOME RESULTS ON STRONG LIMIT THEOREMS, June 1991 (*Air Force and Army*)

Let E be a strict (LB)-space, i.e., a strict inductive limit of separable Banach spaces $E_1 \subset E_2 \subset \cdots$. One of the results proved in this paper is the following. Let X_n, $n \geq 1$ be a sequence of independent identically distributed (*iid*) random variables taking values in E. If the Strong Law of Large Numbers holds for this sequence, i.e., $(1/m)\sum_{i=1}^{m} X_i$, $m \geq 1$ converges almost surely, then there exists $n \geq 1$ such that each X_i takes values in E_n almost surely.

91-11 Jiang Tiefeng, M. Bhaskara Rao, Wang Xiangchen and Li Deli. LAWS OF LARGE NUMBERS AND MODERATE DEVIATIONS FOR STOCHASTIC PROCESSES WITH STATIONARY AND INDEPENDENT INCREMENTS, June 1991 (*Air Force and Army*)

Let $\{X(t);\ t \geq 0\}$ be a stochastic process with stationary and independent increments which has no Gaussian component. Assume that $X(1)$ has a finite moment generating function. Let P_λ be the probability measure of the process $\{Z_\lambda(t);\ 0 \leq t \leq 1\}$, where $Z_\lambda(t) = X(\lambda\alpha[0,\ t])/(\lambda^{1/2}\varphi(\lambda))$, α is a probability measure on $[0,1]$ and $\varphi(\bullet)$ is a positive function such that $\varphi(\lambda) \to \infty$ as $\lambda \to \infty$. We may regard P_λ as a probability measure on $BV[0,1]$, the space of functions of bounded variation on $[0,1]$. In this paper, we establish some results on moderate deviations for $\{P_\lambda;\ \lambda > 0\}$. We also present the Marcinkiewicz-Zygmund type Strong Law of Large Numbers for $\{X(t);\ t \geq 0\}$.

91-13 I. L. Dryden and K. V. Mardia. SIZE AND SHAPE ANALYSIS OF LANDMARK DATA, July 1991 (*Air Force and Army*)

The paper deals with statistical analysis of landmark data using the exact joint distribution theory for the size and shape of planar Gaussian configurations. This is an extension of earlier work which considered marginal shape analysis. Special cases of the size and shape distribution are examined and the isotropic Gaussian model is investigated in particular detail. Various properties are studied, including conditional distributions and a curve of regression. Finally we consider some possible approaches to inference, including exact maximum likelihood estimation of size and shape. A practical biological application is given which investigates the size and shape of mouse vertebrae.

91-14 C. Radhakrishna Rao, Lu Zhang and L. C. Zhao. MULTITARGET ANGLE TRACKING AN ALGORITHM FOR DATA ASSOCIATION, July 1991 (*Army*)

We consider the problem of tracking several targets in terms of the directions of arrival (DOA) of signals using sensor array outputs. The observations in any short interval of time around a given time point τ enable us to estimate the DOA of the targets but not identify the target to which each estimate corresponds. In this paper, we propose an algorithm to set up such a correspondence between estimates and targets at each time point in a sequential manner starting from an initial position of the targets. Once this is done, the trajectories of the different targets can be obtained by plotting the estimates against time.

91-15 Z. D. Bai and Y. Wu. LIMITING BEHAVIOR OF M-ESTIMATORS OF REGRESSION COEFFICIENTS IN HIGH DIMENSIONAL LINEAR MODELS. I. Scale-dependent case, July 1991 (*Air Force* and *Army*)

Asymptotics of M-estimators of the regression coefficients in linear models (both scale variant and scale invariant) when the number of regression coefficients tends to infinity as the sample size increases are investigated. The main purpose of this study is to establish the asymptotic properties under weaker conditions than usually assumed, especially to relax the restrictions on the order of the dimension. Also, the conditions assumed and the results obtained seem easy to be extended to the multivariate linear models. In the first part of the paper, the asymptotic behavior of the ordinary (i.e., not scale-invariant) M-estimates are considered.

91-17 Nandini Kannan and Debasis Kundu. ON MODIFIED EVLP AND ML METHODS FOR ESTIMATING SUPERIMPOSED EXPONENTIAL SIGNALS, July 1991 (*Army*)

We consider the multiple sinusoidal model for signals and provide modified ML and EVLP estimates when the damping factor is not present. The solution in the general case depends on the roots of a polynomial equation, whose coefficients are estimated from observed data. When the damping factor is absent, the coefficients exhibit a certain symmetry. Under these conditions satisfactory algorithms have been developed to estimate the coefficients. A simulation study has been conducted to compare these and existing methods of estimation.

91-19 K. V. Mardia, J. T. Kent and A. N. Walder. STATISTICAL SHAPE MODELS IN IMAGE ANALYSIS, August 1991 (*Army*)

We discuss several models for shapes in the plane based on the distributions of landmarks about an underlying template. The motivation for these models includes Markov random fields and thin plates splines. These models are used as priors in a Bayesian framework to reconstruct a shape from a digital image. An example is given based on the human hand

91-20 Colin R. Goodall amd Kanti V. Mardia. MULTIVARIATE ASPECTS OF SHAPE THEORY, August 1991 (*Army*)

We place shape theory in the setting of noncentral multivariate analysis, and thus provide a comprehensive view of shape distributions when landmark coordinates are Gaussian distributed. This work allows the statistical analysis of shape to be carried out using standard techniques of multivariate analysis. The two components of the QR decomposition of the centered matrix of landmark coordinates are the lower-triangular matrix representing size-and-shape and a matrix with orthonormal columns. The size-and-shape is obtained by integrating away this orthogonal component. As the orthogonal component is usually square, an important distinction must be made between shape densities

with reflection included, or specifically excluded. The shape of the figure is obtained by integrating away the size, or matrix norm, from the size-and-shape component. Thus there are fundamental differences between multivariate analysis and shape theory, and shape theory requires new results in multivariate distribution theory. Our results include an almost complete derivation of size-and-shape densities and shape densities in any dimension. These include the results for planar figures worked out very recently by several authors. Any practical application of shape theory depends on a system of shape coordinates. We give a general algorithm based on the QR decomposition, that includes polar shape coordinates, and Kendall and Bookstein coordinates generalized to arbitrary dimension as special cases. Through an invariant our distributional results are available in any of these coordinate systems.

91-22 C. Radhakrishna Rao and Z. J. Liu. MULTIVARIATE ANALYSIS UNDER M-ESTIMATION THEORY USING A CONVEX DISCREPANCY FUNCTION, September 1991 (*Air Force* and *Army*)

We consider a general multivariate linear regression model $Y_i = X_i'\beta+\epsilon_i$, $i = 1,\ldots,n$, where Y_i and ϵ_i are p-vector random variables, X_i is a q×p design matrix and β is a q-vector of unknown parameters. We develop a general theory for the estimation of β and tests of hypotheses on β using the concepts of M-estimation. Specifically, we consider the estimation of β by minimizing

$$\sum_1^n \rho(Y_i-X_i'\beta)$$

where the discrepancy function, ρ, is convex. The special case of the MANOVA model, where X_i has a simple structure, is considered in some detail.

91-23 C. Radhakrishna Rao. R. A. FISHER: THE FOUNDER OF MODERN STATISTICS, October 1991 (*Army*)

Before the beginning of the present century, statistics meant observed data and descriptive summary figures such as means, variances, indices, etc., computed from data. With the introduction of the chi-square test for goodness of fit (specification) by Karl Pearson (1900) and the t test by Gosset (Student, 1908) for drawing inference on the mean of a normal population, statistics started acquiring new meaning as a method of processing data to determine the amount of uncertainty in various generalizations we may make from observed data (sample) to the source of the data (population).

The major steps which led to the establishment and recognition of statistics as a separate scientific discipline and an inevitable tool

in improving natural knowledge were taken by R. A. Fisher during the decade 1915-1925. Most of the concepts and methods introduced by Fisher are fundamental and continue to provide the framework for the discussion of statistical theoery. Fisher's work is monumental both in richness and variety of ideas and provided the inspiration for phenomenal developments in statistical methodology for applications in all areas of human endeavour during the last seventy-five years.

Some of Fisher's pioneering works have raised bitter controversies which still continue. These controversies have indeed helped in highlighting the intrinsic difficulties in inductive reasoning and seeking for refinements in statistical methodology.

91-26 Tiefeng Jiang, Xiangchen Wang and M. Bhaskara Rao. MODERATE DEVIATIONS FOR SOME WEAKLY DEPENDENT RANDOM PROCESSES, November 1991 (*Air Force* and *Army*)

In this paper we compute moderate deviations for two classes of weakly dependent processes, namely, moving averages of independent identically distributed random variables and Poisson center cluster random measures.

91-27 C. R. Rao, Lu Zhang and L. C. Zhao. MULTIPLE TARGET ANGLE TRACKING USING SENSOR ARRAY OUTPUTS, November 1991 (*Army*)

We consider the problem of tracking the directions of arrival of signals from a number of objects using sensor array outputs. An algorithm is developed for estimating the angles associated with different objects directly, avoiding data association methods. Simulation results indicate that the proposed method works well in practice.

91-28 P.D.L. Constable and K. V. Mardia. ON SIZE AND SHAPE ANALYSIS OF PALMAR INTERDIGITAL AREAS, December 1991 (*Air Force* and *Army*)

As pointed out by Hauser and Abraham (1985), dermatoglyphic methodology investigates either shape or size rather than both variables simultaneously. Hauser and Abraham (1985) have given a method to analyse shape and size simultaneously of the third palmar interdigital area. We describe the selections of their shape variables analytically and indicate how it can fit into a general framework of size and shape analysis. Their data are reanalysed and we describe some advantages of the choice of their variables. However their size and shape variables are highly correlated.

91-29 C. Radhakrishna Rao, Lu Zhang and L. C. Zhao. A NOTE ON MULTITARGET ANGLE TRACKING, December 1991 (*Army*)

In a previous paper (1991a) the authors considered the problem of tracking several targets in terms of the directions of arrival (DOA) of signals uisng sensor array outputs. An algorithm was developed for data association. In this note, we consider the problem of estimating the DOA's by a new method, which seems to provide satisfactory estimates. The algorithm developed for data association in the previous paper is used on the new estimates for computing the trajectories of the different targets.

91-33 Debasis Kundu. DETECTING THE NUMBER OF SIGNALS FOR UNDAMPED EXPONENTIAL MODELS USING INFORMATION THEORETIC CRITERION, December 1991 (*Army*)

We consider the estimation of the number of signals in exponential signals models. We use different Information Theoretic Criteria to detect the number of signals and compare its small sample properties by Monte Carlo Simulation study.

91-34 Debasis Kundu. A MODIFIED PRONY ALGORITHM FOR SUM OF DAMPED OR UNDAMPED EXPONENTIAL SIGNALS, December 1991 (*Army*)

Osborne (1975) introduces a modified Prony algorithm for fitting sum of exponential to the real data sets. Smyth (1985) and Osborne and Smyth (1990) investigate the stability of the algorithm. They show by numerical comparison that the modified Prony algorithm works better than any other standard non-linear regression algorithm. In this paper we generalize the concept of Osborne (1975) to the complex parameters case, which is very important in signal processing. We investigate its stability and show by numerical experiments that the modified Prony algorithm works very well for complex exponentials in complex noise also.

91-37 Z. J. Liu. ASYMPTOTIC EFFICIENCY OF ONE WAY ANALYSIS OF RAO'S QUADRATIC ENTROPY, December 1991 (*Air Force* and *Army*)

We investigate the asymptotic efficiency of the analysis of Rao's quadratic entropy (ANOQE) statistics which is a generalization of the classical analysis of variance (ANOVA) statistics. The study of the asymptotic distribution and the bootstrap approximation of ANOQE statistics has been done by Rao and Liu. In this paper the Bahadur's efficiency of the ANOQE statistics is derived and some examples are presented.

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92-04 C. R. Rao, C. R. Sastry and B. Zhou. TRACKING THE DIRECTION OF ARRIVAL OF MULTIPLE MOVING TARGETS, June 1992 (*Army*)

In this work we focus on the problem of tracking the direction-of-arrival (DOA) of multiple moving targets. The targets are assumed to be moving with constant accelerations subject to minor random perturbations and emitting narrow band signals that impinge on an array of passive sensors. Estimates of the DOA vector of the targets are obtained from the sensor data based on the maximum likelihood (ML) principle in such a way that the association between the estimates made at different time points is maintained. At each stage, the current ML estimates of DOA are treated as measurements and refined via a Kalman filter and tracking is accomplished without the need to perform unduly heavy computations. An efficient strategy for dealing with closed by spaced targets is also presented. Finally, the performance of our tracking agorithm is illustrated via computer simulations.

92-12 C. R. Rao, M. S. Srivastava and Yanhong Wu. SOME ASPECTS OF QUALITY CONTROL METHODS, July 1992 (*Air Force* and *Army*)

This paper reviews quality control methods when the quality characteristic can be measured by a continuous variable. There are three aspects of quality control: 1) Statistical Process Control (SPC), 2) Automatic Process Control (APC), and 3) Lot Inspection (LI). The first two methods are called on-line quality control methods and are applied to control the quality of a production process while in operation. The third method is usually used after production and uses a sampling device to inspect the finished products for defectives.

92-14 Z. D. Bai C. R. Rao Y. H. Wu. A NOTE ON M-ESTIMATION OF MULTIVARIATE LINEAR REGRESSION, June 1992 (*Army*)

M-estimates of the parameters in a linear model are obtained by minimizing the sum of functions of residuals. The function called the "discrepancy function" and denoted by ρ, is chosen to provide robust estimates. In this paper we suggest the choice, $\rho = \rho_1 - \rho_2$, where ρ_1 and ρ_2 are convex functions. By choosing ρ_1 and ρ_2 suitably, we can generate a wide variety of discrepancy functions which can be used in practice. There is an added advantage that the conditions placed on ρ_1 and ρ_2 and hence on ρ are much milder than those considered in the literature on M-estimation.

92-16 Gutti Jogesh Babu and C. Radhakrishna Rao. BOOTSTRAP METHODOLOGY, June 1992 (*Air Force* and *Army*)

Theoretical and computational aspects of the bootstrap methodology are reviewed. Consistency and asymptotic accuracy of the bootstrap methods are discussed in detail. The methods for non-smooth functions, significance tests, construction of confidence intervals, are also reviewed. A brief discussion of bootstrap methods for censored, sample survey, regression, autoregressive and time series models is also given. Situations where the bootstrap method fails are indicated. A summary of Bayesian bootstrap methods is given.

92-17 Z. D. Bai, Z. J. Liu and C. Radhakrishna Rao. ON THE STRONG CONSISTENCY OF M-ESTIMATES IN LINEAR MODELS UNDER A GENERAL DISCREPANCY FUNCTION, June 1992 (*Air Force* and *Army*)

Huber (1964) considered the estimation of the parameter β in the linear model $Y_i = X_i'\beta+\epsilon_i$, $i = 1,\ldots,n$, by minimizing $\Sigma\rho(Y_i-X_i'\beta)$, where ρ is a suitably chosen "discrepancy function". Properties of such estimates known as M-estimates have been considered by several authors for particular choices of ρ. In this paper we consider a general ρ which in non-increasing in $(-\infty,0]$ and non-decreasing $[0,\infty)$ and establish strong consistency of the M-estimate. Such a choice of ρ covers most of the cases considered by earlier writer.

92-18 C. R. Rao, C. R. Sastry and Bin Zhou. SOME RECENT CONTRIBUTIONS TO MULTITARGET TRACKING, June 1992 (*Army*)

In this paper, the most recent contributions in multitarget tracking are reviewed. The areas covered include efficient algorithms for data association and estimation, and tracking of the directions of arrival (DOA) using data from an array of sensors. An introduction to the symmetric measurement equation (SME) filter approach to multitarget tracking is also provided. Finally, some suggestions are made for future research.

92-19 C. R. Rao. SOME STATISTICAL PROBLEMS IN MULTITARGET TRACKING, June 1992 (*Army*)

In tracking a single target, dynamical and observational equations are used to estimate, via Kalman filter, the positional coordinates of the target at different time points, and the trajectory is obtained by joining the successive positional coordinates over time. In the case of multiple targets, we generally do not have separate observations on each target. At any time point, we have a set of measurements without the information as to which measurement belongs to which object. The statistical problem is that of estimating the positional coordinates of the individual targets from such mixed up data using the information provided by the dynamical equations and the previous estimates.

92-23 Z. D. Bai and Y. Wu. LIMITING BEHAVIOR OF M-ESTIMATORS OF COEFFICIET VECTOR IN HIGH DIMENSIONAL REGRESSION MODELS. II. SCALE-INVARIANT CASE, July 1992 (*Army*)

Consider the linear model

$$y_{in} = x_{in}'\theta_{no} + e_{in}, \quad 1 \le i \le n$$

where x_{in} is a sequence of known p-vectors, $\theta_{no} \in R^p$ is an unknown vector of regression coefficients, $\{e_{in}\}$ is a sequence of iid random errors and $p = p_n$ tends to infinity as $n \to \infty$.

Let ρ be a discrepancy function and $\hat{\theta}_n$ and $\hat{\sigma}_n$ be M-estimates obtained by minimizing

$$\sum_{i=1}^{n} \rho[(\sigma^{-1}(y_{in} - x_{in}'\theta)) + \log \sigma].$$

Further, let $\tilde{\theta}_n$ be an M-estimate obtained by minimizing

$$\sum_{i=1}^{n} \rho(\tilde{\sigma}_n^{-1}(y_{in} - x_{in}'\theta)).$$

The properties of $\hat{\theta}_n$ and $\tilde{\theta}_n$ are studied as $n \to \infty$.

92-24 Min Deng. DIFFERENTIAL GEOMETRIC STRUCTURE OF A STATISTICAL MODEL, August 1992 (*Army*)

In this paper be introudce a differential geometric constructions in probability spaces, which is different from the information metric introduced by Rao. The new Riemann metric plays a very important role in discussing curvatures and Fisher information matrix. It is suggested that the new Riemann metric may yield results in statistical inference. We also show that the new Riemann metric is asymptotically equivalent to the Riemann metric proposed by Rao (1945).

92-25 Min Deng. CURVATURES AND THE INFORMATION. MATRRIX, August 1992 (*Army*)

In this paper, we introduce the concept of Gauss curvature of a statistical model. In this connection, a natural question asked. Does Gauss curvature and Fisher information matrix have any relationship? The paper provides an answer to this question. Also other curvatures of a statistical model are introduced and the relationshhips between these curvatures and Fisher informatio are established.

92-26 Min Deng. CURVATURE AND SECOND ORDER ASYMPTOTIC PROPERTIES OF THE GENERALIZED BAYES ESTIMATORS FOR THE LOCATION FAMILY, August 1992 (*Army*)

In this paper, we find the second order asymptotic density function of the generalized Bayes estimator for a truncated location family, and give the upper bound for concentration probability for 3/2-th order asymptotically median unbiased (AMU) estimators. It is pointed out that the curvature of this family plays a very important role in the second order asymptotic properties of the generalized Bayes estimators for a truncated location family. Also one of the results obtained here has been approximation than Akahira's result and another is equivalent to his. Finally, asymptotic generalized Bayes estimators up to the second order for the two parameters truncated location family are derived.

92-27 Min Deng. INFORMATION LOSS IN ESTIMATION OF PARAMETERS, August 1992 (*Army*)

In this paper, we investigate the asymptotic loss of information matrix in estimation of parametes in a multiparameter family. This extends the results of Fisher, Rao and Efron on the loss of information in the one parameter case to several parameters. In particular, we consider two-parameter family of distributions and computer the information loss for a number of methods of estimation like maximum likelihood minimum chisquare, etc.

92-28 C. Radhakrishna Rao and Bin Zhou. CLOSED FORM SOLUTION TO THE ESTIMATES OF DIRECTIONS OF ARRIVAL USING DATA FROM AN ARRAY OF SENSORS, August 1992 (*Army*)

Closed form solution to the estimates of directions of arrival (DOAs) of signals is obtained through solving a polynomial equation of degree equal to the number of distinct signals. The coefficients of the polynomial are estimated by expressing the condition that the vector of coefficients augmented by zeroes belongs to the noise subspace or is orthogonal to the signal subspace.

92-30 Debasis Kundu and Nandini Kannan. ESTIMATION OF DIRECTION OF ARRIVAL OF SIGNALS, August 1992 (*Army*)

In this paper, the authors proposed some modifications to the existing methods of estimatioin of the unknown directions of arrival of signals from several sources using both the data vectors and its conjugate. The strong consistency of the estimators have been established. Simulation results that illustrate the performance of the different methods are presented.

92-31 C. Radhakrishna Rao. CURRENT TRENDS OF RESEARCH IN STATISTICS SMALL SAMPLE ASYMPTOTIC, RESAMPLING TECHNIQUES AND ROBUSTNESS, June 1992 (*Army*)

In many statistical problems, it is often difficult to evaluate the exact distribution of a statistic. In such cases we resort to asymptotic methods. The paper surveys some recent results on asymptotic expansions of the distribution of a statistic, with successive terms providing improvement in accuracy but decreasing in magnitudes with orders of successive powers of $1/n$ or $1/\sqrt{n}$, where n is the sample size. It also describes the methods of jackknife and bootstrap for making approximate computations of some distributional aspects of estimators and test criteria. Recent work on robust inference is briefly mentioned.

92-32 Rahul Mukerjee. RAO'S SCORE TEST: RECENT ASYMPTOTIC RESULTS, September 1992 (*Army*)

This article reviews some recent asymptotic results on Rao's score test. A third-order optimality property of this test, under contiguous alternatives, has been discussed. Some approaches towards a Bartlett-type adjustment for Rao's statistic have been summarized. A few related open problems are also indicated.